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Effective elastic moduli of porous solids

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The principles of continuum mechanics can be extended to porous solids only if the effective moduli are known. Although the effective bulk modulus has already been determined by approximating the geometry of a porous solid to be a hollow sphere, bounds could only be established for the other moduli. This problem of indeterminacy of the moduli is solved in this study using a particular model from the variation of the effective Poisson's ratio. In addition to this, the results are extended for the hollow sphere to real geometry by introducing a porosity-dependent factor. These results are compared with experimental data and the agreement is found to be good. As the effective Poisson's ratio cannot be determined accurately using experiments, the derived equation is verified using finite element analysis.

1. Introduction

Porous solids find many engineering applications. These range from acoustic absorption to heat shields in re-entry vehicles. Even in nature many of the solids are porous making them optimally designed for the load they carry and the environment in which they exist.

As porous solids by their very nature are heterogeneous, analytical procedures available for treating homogeneous solids as continuum materials are not directly applicable. However, if pores are distributed with respect to their size, shape and spatial distribution in a statistically random fashion, it is possible to treat the porous solid as a continuum material taking into account the various effective properties which depend on the properties of the corresponding dense solid and the porosity.

Mechanical properties of two-phase solids have been studied both theoretically and experimentally. Most of these [1–7], however, involved studies on two-phase materials with each phase having different elastic moduli. Dewey [1] and Paul [2] established the elastic constants of two-phase materials where one of the phases was long fibres distributed in a matrix. Eshelby [3] and later Hashin [4] and Hashin and Shtrikman [5] studied the elastic properties where the second phase is distributed in a statistically random fashion. Hill [6] and Budiansky [7] applied the concepts of self consistency of deformation of composite materials for determining the elastic constants. Among the many authors, only Mackenzie [8] studied the elastic properties, exclusively of porous solids, using a hollow sphere model.

Even though many investigators determined the elastic moduli of porous solids by treating them as a particular case of two-phase solids, where one of the phases is a void, we believe that this temptation must be resisted for the following reason. Although the effective bulk modulus is determinable using statistical continuum theory, it is possible to establish only bounds for the effective shear modulus. The gap between the bounds increases with the difference in the elastic properties of the phases. Therefore these bonds for porous solids will have no meaning. The model used for such calculations, i.e. an assemblage of non-interacting spheres with concentric spherical inclusions, is also strictly not valid when the difference between the elastic properties of the phases is large. We shall prove this point later in this paper.

In this paper we develop analytical procedures for determining the effective elastic constants of porous solids with pores distributed randomly. We begin this analysis with equations developed by Hashin and Shtrikman [4, 5] for multiphase materials. As the effective shear modulus is not determinable by this method, we shall use a variation of the effective Poisson's ratio as a parameter for determining the shear modulus. We shall compare the results of these calculations with the experimentally measured parameters. As the experimentally determined effective Poisson's ratios are generally not accurate, we shall compare the results of the calculations with those determined in a study where a finite element analysis was carried out on a geometry with spherical voids distributed in a material matrix.

2. Determination of the effective bulk modulus

The derivation begins with the following general equation given by Hashin and Shtrikman [4, 5]

$$\overline{\sigma_{ii}} = 3K^* \, \overline{\varepsilon_{ii}} \tag{1}$$

where $\overline{\sigma_{ii}}$ and $\overline{\varepsilon_{ii}}$ are volume averages of the hydrostatic stress and strain, respectively. We will first establish the effective bulk modulus of a hollow sphere and later extend it to multipore geometry. The averages are

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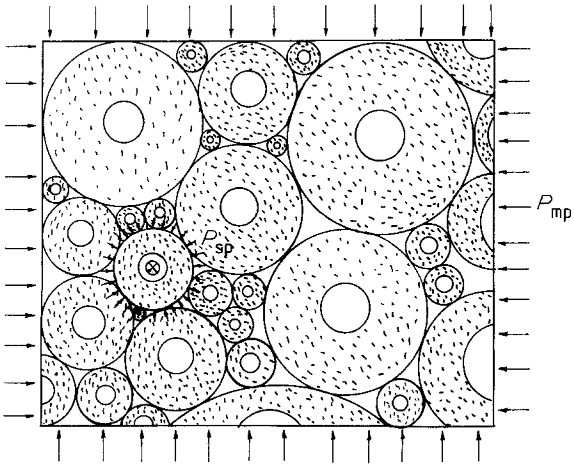


Figure 1 Assemby of hollow spheres representing real geometry.

calculated over the entire volume of the hollow sphere comprising both the void and the material phase. The stress and strain distributions in the material phase, which can be determined easily, are given in Appendix 1. In the void the stress is zero but not the strain. The average volumetric strain in the void can be determined directly as the void shrinkage is known from the displacement at the inner surface of the hollow sphere. Following the above procedure we obtain the effective bulk modulus as

$$K^* = K(1 - \theta)/(1 + b_k\theta) \quad (2)$$

where $b_k = (1 + \mu)/[2(1 - 2\mu)]$, K and μ are the bulk modulus and the Poisson's ratio of the corresponding dense material. Details of this derivation are given in Appendix 2. This is now extended to an assemblage of hollow spheres as shown in Fig. 1, in which each sphere, though different in diameter, has the same void fraction. This is similar to the procedure adopted by the earlier workers as well. However, they assumed that the pressure exerted on the surface of the sphere is the same as that exerted on the multipore geometry. This approximation may be acceptable in the case of two-phase materials where the moduli of the phases do not differ much, but in the case of porous solids there is a definite need to establish the correction factor and the procedure we adopted is given below.

Let P_{sp} be the pressure acting on the hollow sphere extracted from the multipore geometry and P_{mp} be the pressure acting on the multipore geometry itself. Pressure being zero in the pore volume, the average pressure in the material volume is $P_{mp}/(1 - \theta)$ where θ is the porosity. Therefore

$$P_{sp} = P_{mp}/(1 - \theta) \quad (3)$$

As the volume strains of both the hollow sphere and the assemblage are equal

$$K^*_{mp}/K^*_{sp} = P_{mp}/P_{sp} = (1 - \theta) \quad (4)$$

Combining Equations 2 and 4 we obtain

$$K^* = K(1 - \theta)^2/(1 + b_k\theta) \quad (5)$$

(From here on, K^*_{mp} will be denoted by K^*). This equation satisfies all the limiting conditions of zero porosity and a totally porous solid. The variation of the effective bulk modulus with porosity, for a few typical values of the dense material Poisson's ratio, is given in Fig. 2.

3. Determination of the effective shear modulus

Similar to Equation 1, the basic equation [4, 5] representing the effective shear modulus is expressed in the form

$$\overline{S_{ij}} = 2G^* \, \overline{e_{ij}} \quad (6)$$

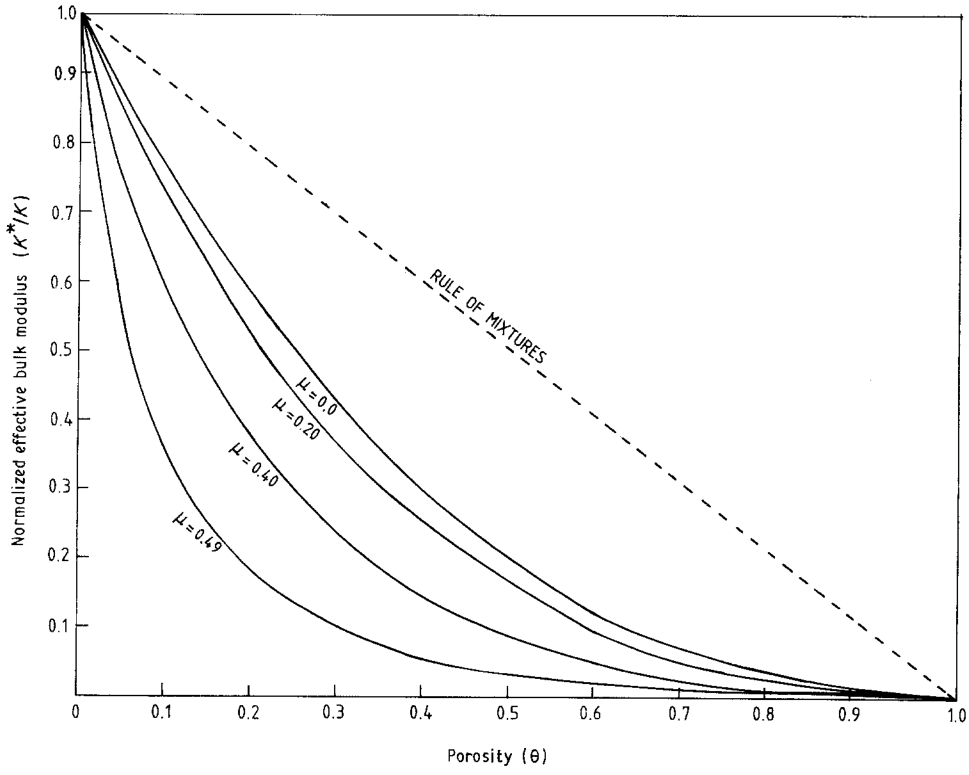


Figure 2 Variation of the effective bulk modulus with porosity.

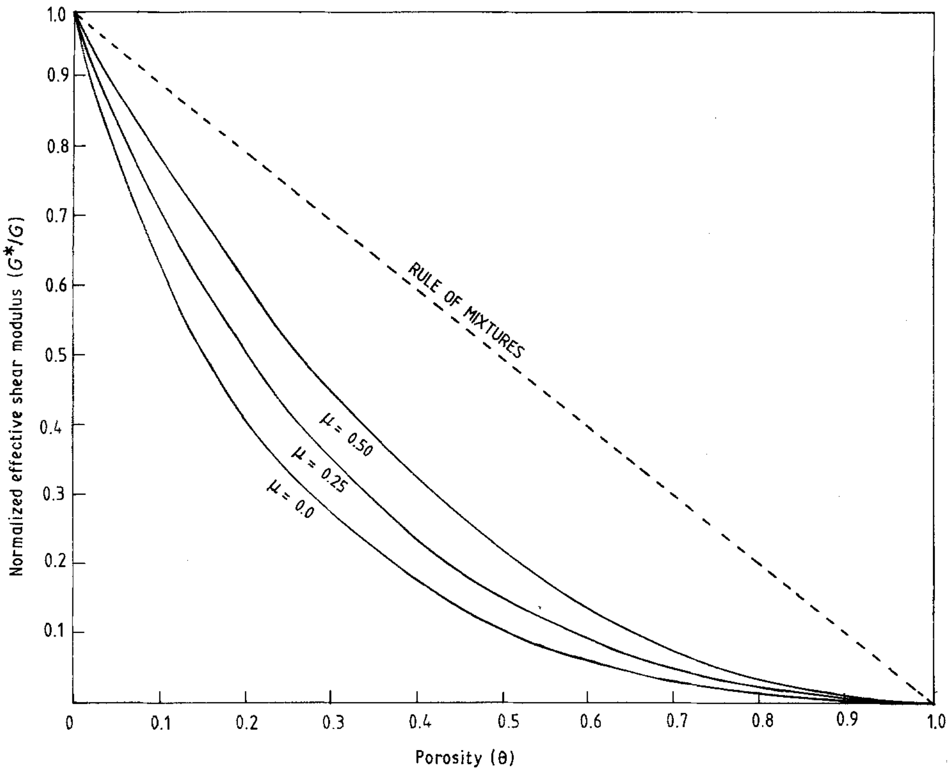


Figure 3 Variation of the effective shear modulus with porosity. $G/G^* = (1 - \theta)^2/(1 - b_g\theta)$ where $b_g = (11 - 19)/4\ (1 +)$.

where $\overline{S_{ij}}$ and $\overline{e_{ij}}$ are volume averages of the deviatoric stress and strain, respectively. The derivation of the effective shear modulus is relatively more difficult than that of the effective bulk modulus. This is because the average deviatoric strain in the void cannot be determined directly as could be done in the case of the average volumetric strain. Therefore, the average deviatoric strain in the void essentially remains unknown in the derivation and requires an additional equation to resolve it; otherwise only bounds may be established for the effective shear modulus. To find this additional equation we propose a model for the variation of the effective Poisson's ratio with the porosity as follows.

(a) For a completely porous solid ($\theta = 1$) the effective Poisson's ratio is a constant, independent of the material. (Let this constant be k).

(b) In this case of the porous solids for which the Poisson's ratio at its fully dense state is k, the effective Poisson's ratio does not vary with porosity.

With this model it becomes possible to determine the effective shear modulus of the hollow sphere in terms of shear modulus of the corresponding dense solid, the porosity and k. To extend this to multipore geometry the effective shear modulus is multiplied by the factor $(1 - \theta)$ derived in the previous section. Also, using the experimental results, we found k to be 1/4. Finally, we find the effective shear modulus to be

$$G^* = G(1 - \theta)^2/(1 + b_g\theta) \quad (7)$$

where $b_g = (11 - 19\mu)/\{4(1 + \mu)\}$, and G and μ are the shear modulus and the Poisson's ratio of the corresponding dense solid, respectively. The detailed derivation is provided in Appendix 3. This equation also satisfies all the limiting conditions like Equation 5. The variation of the effective shear modulus with porosity, for a few typical values of the corresponding dense material Poisson's ratio, is given in Fig. 3.

4. Determination of the effective Young's modulus and Poisson's ratio

Determining the effective Young's modulus and the effective Poisson's ratio is only a matter of using the inter-relationships between different moduli, because the effective bulk and shear moduli have already been established. The effective Young's modulus is given by

$$E^* = E(1 - \theta)^2/(1 + b_\theta\theta) \quad (8)$$

where $b_\theta = 2$ to 3μ. The effective Poisson's ratio is

$$\mu^* = \frac{(1/4)(4\mu + 3\theta - 7\mu\theta)}{(1 + 2\theta - 3\mu\theta)} \quad (9)$$

where E and μ are the Young's modulus and the Poisson's ratio of the corresponding dense material. The variations of the effective Young's modulus and the effective Poisson's ratio with porosity are shown in Figs 4 and 5.

5. Experimental validation of the analysis

A number of experimental results are available on the variation of effective elastic moduli with porosity. We chose the experimental results on porous MgO [9], $MgAl_2O_3$ [10] and Sm_2O_3 [11] because they represent a good variation in their dense material Poisson's ratios (the dense material Poisson's ratio of MgO is 0.18, that of $MgAl_2O_3$ is 0.268 and that of Sm_2O_3 is 0.324). The experimental results are plotted with the values calculated using the derived equations and are shown in Figs 6a to c.

Dean and Lopez [12] compiled experimental data on various porous materials with porosity ranging

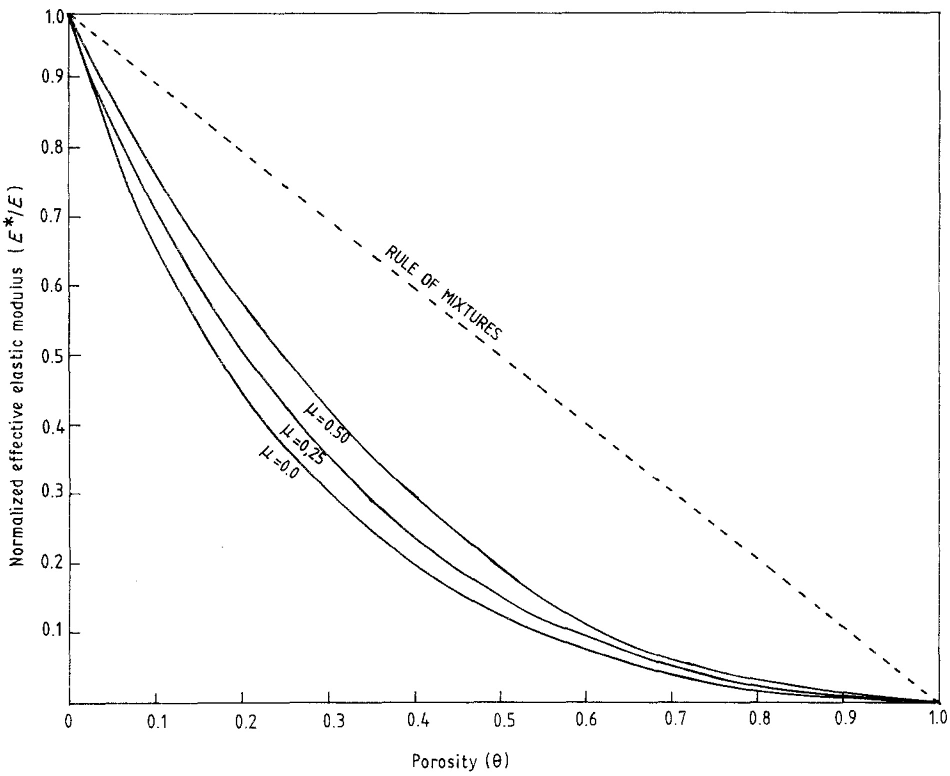


Figure 4 Variation of the effective Young's modulus with porosity. $E^*/E = (1 - \theta)^2/(1 + b_e\theta)$ where $b_e = 2 - 3$.

from zero to about 35%, in order to find the best form of empirical equation to fit the variation of the effective moduli with porosity. They found a linear equation to be the most suitable one in this porosity range. In order to compare these results with our derived equations, we have also used linear approximation of the derived equation in the relevant porosity range. The agreement between the experimental results and the derived equations is excellent.

Experimental validation of the effective Poisson's ratio is relatively difficult because of the inherent inaccuracy in determining μ^* through experimental values of E^* and G^*. However, some qualitative statements made on the variation of μ^* with porosity support the derived equations. Manning *et al.* [13] report a decrease in the effective Poisson's ratio of yttrium oxide, holmium oxide and erbium oxide with porosity. This, according to the derived equations is to be expected, because their dense material Poisson's ratios are above 0.25. Coble and Kingery [14] also make an interesting experimental observation that the effective Poisson's ratio of Al_2O_3 remains nearly constant at 0.25 with increasing porosity. This corroborates the model proposed for the variation of the effective Poisson's ratio with porosity in this analysis. Similarly, the experimental results of Saga and Scheriber [15] on MgO, which has a Poisson's ratio less than 0.25 in its fully dense state, show an increase in μ^* with porosity as expected.

Another interesting validation of the effective Poisson's ratio could be made by comparing the results of the derived equations with those of a detailed finite element analysis done on a pore geometry. A finite element mesh was created with a set of concentric circles as shown in Fig. 7. By assigning zero modulus to elements of a few circles, the desired level of porosity was incorporated in the geometry. We analysed three such variations with porosities of 0.17,

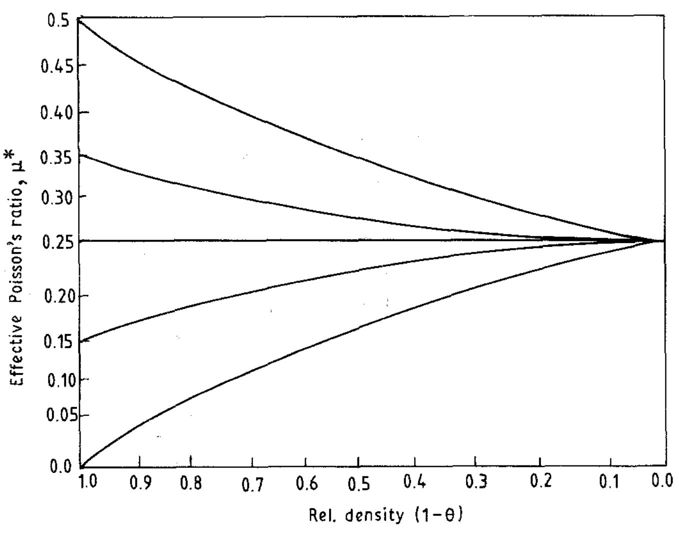
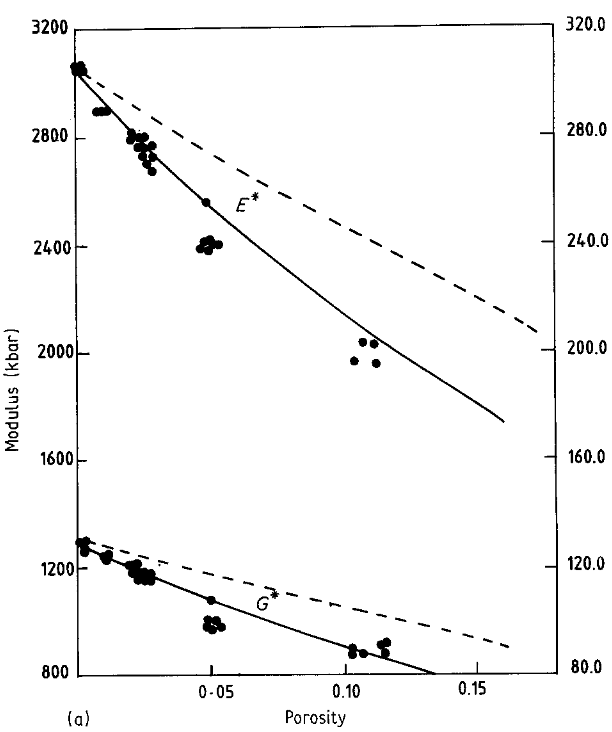
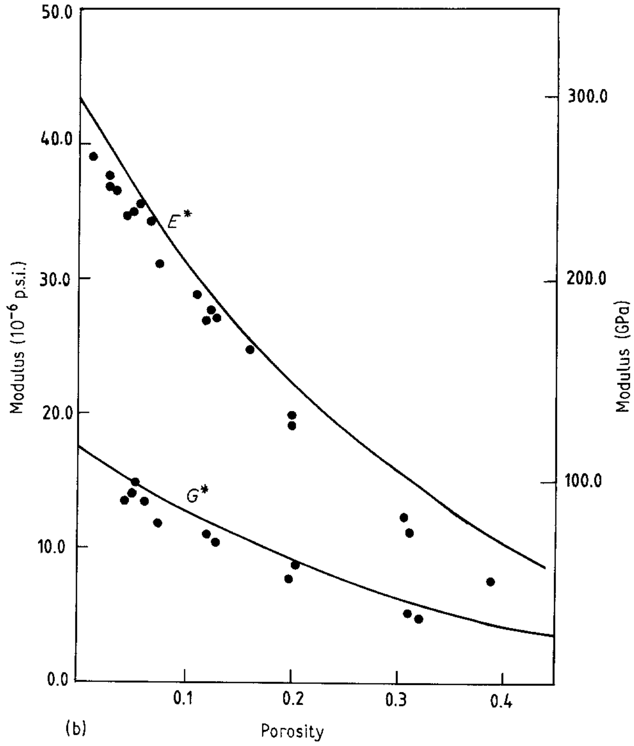


Figure 5 Variation of the effective Poisson's ratio with porosity. $\mu^* = (4\mu + 3\theta - 7\mu\theta)/[4(1 + 2\theta - 3\mu\theta)]$.





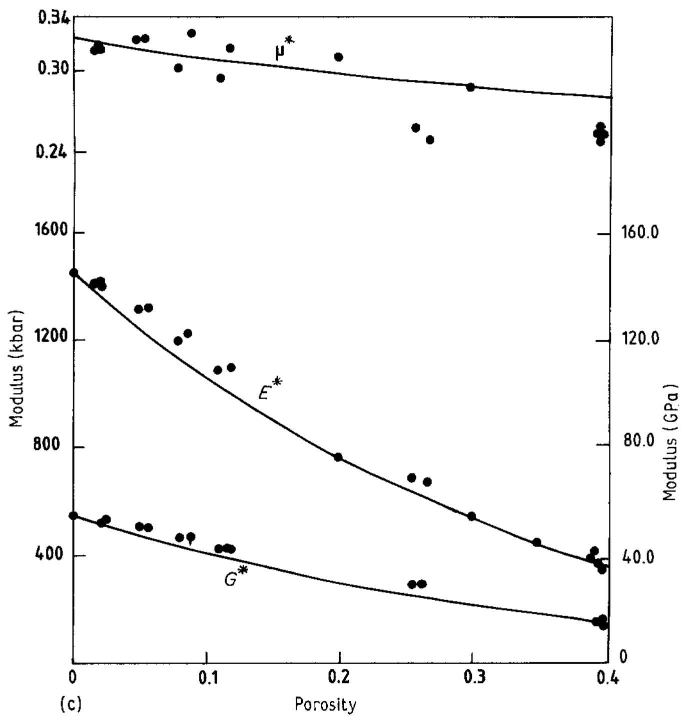


Figure 6 Validation of the derived equation for the effective moduli using experimental data. (a) MgO, E = 3061 kbar (310 GPa), G = 1300 kbar (131.7 GPa), μ = 0.18, (- - -) average of Hashin and Shtrikman's bounds [5], (●) experimental data [9], (——) predicted variation. (b) $MgAl_2O_3$, $E = 43.4 \times 10^6$ p.s.i. (299.2 GPa), $G = 17.2 \times 10^6$ p.s.i. (118.6 GPa), μ = 0.268, (●) experimental data [10], (——) predicted variation. (c) Sm_2O_3, E = 1450 kpar (146.9 GPa), G = 547.5 kbar (55.47 GPa), μ = 0.3245, (●) experimental data [11], (——) predicted variation.

0.33 and 0.57. In each case a typical elastic longitudinal strain was imposed and the average lateral strain was computed for five different values of dense material Poisson's ratio. The results of this analysis are shown in Fig. 8 and an excellent agreement between the derived equations and the FEM results can be seen.

6. Conclusion

Equations have been derived for the effective elastic moduli and Poisson's ratio of porous solids using the principle of statistical continuum mechanics. All these moduli can be described by a general equation

$$M^* = M(1 - \theta)^2/(1 + b_m\theta) \quad (10)$$

where M^* is the effective modulus, M the corresponding dense material modulus and θ the porosity. The term b_m depends on the concerned modulus and is a function of only the dense material Poisson's ratio. The effective Poisson's ratio is given by

$$\mu^* = (1/4)(4\mu + 3\theta - 7\mu\theta)/(1 + 2\theta - 3\mu\theta) \quad (11)$$

Published experimental results on the variations of effective elastic moduli with porosity agree well with the derived equations.

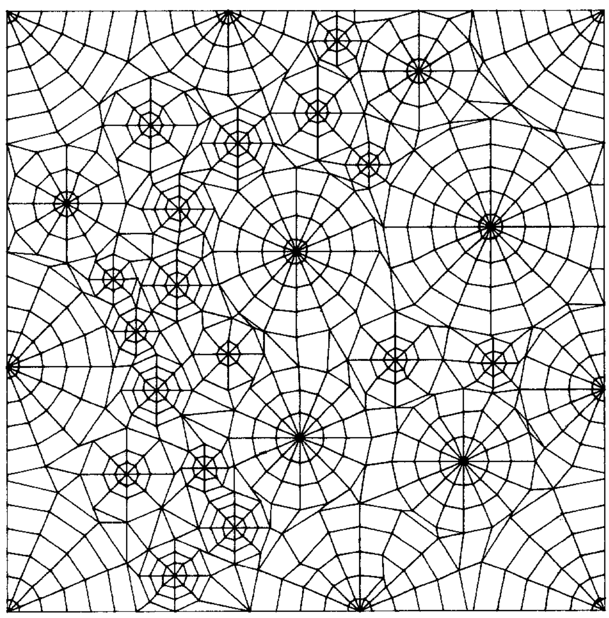


Figure 7 Master mesh for the finite element analysis.

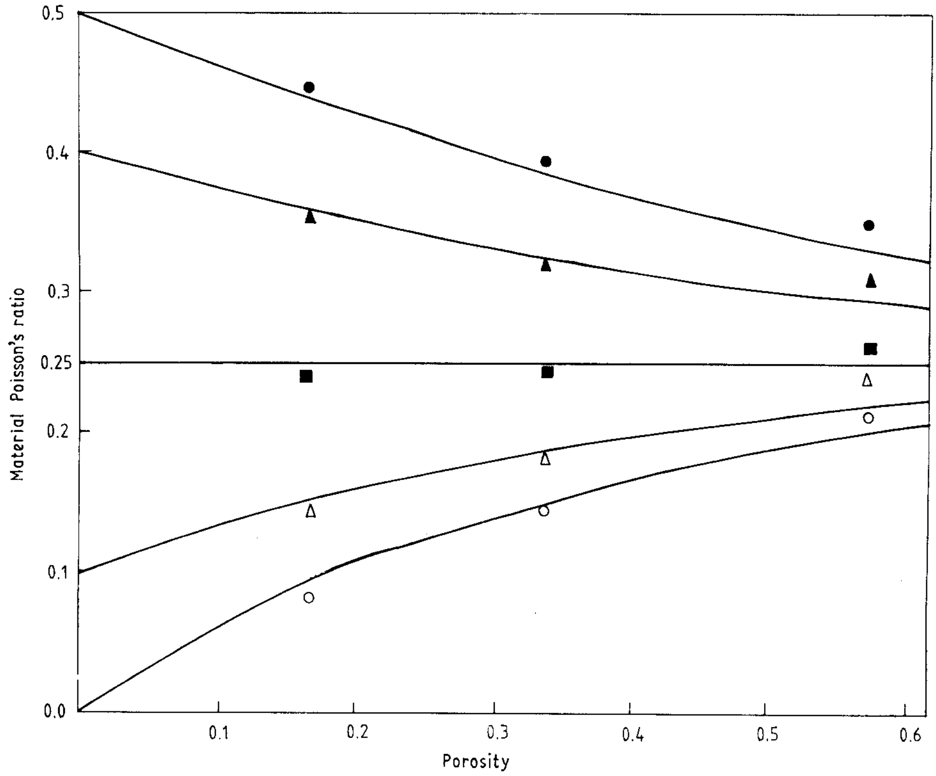


Figure 8 Verification of the derived equation for the effective Poisson's ratio using the results of the finite element analysis. (——) Predicted value using Equation 12. FEM results: μ = (●) 0.495, (▲) 0.400, (■) 0.250, (△) 0.100, (○) 0.000.

Appendix 1. Stress–strain distribution in a thick-walled hollow sphere subjected to external pressure

Consider a hollow sphere of uniform wall thickness (Fig. A1) with external and internal radii a and b, respectively. When the sphere is subjected to a uniform external pressure, P, the radial and tangential stresses (σ_r and σ_t) developed at a distance r from the centre of the sphere are given as

$$\sigma_r = -P(1-\theta')/(1-\theta) \quad \text{(A1)}$$

$$\sigma_t = -P(1+\theta'/2)/(1-\theta) \quad \text{(A2)}$$

where $\theta' = b^3/r^3$, and $\theta = b^3/a^3$. The corresponding

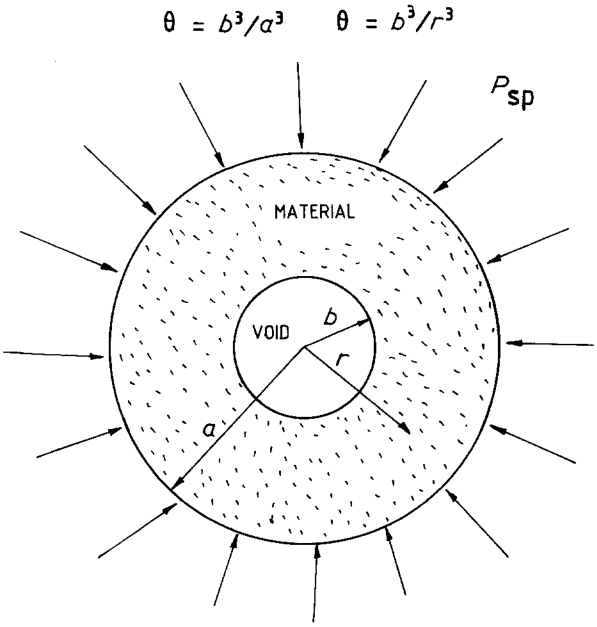


Figure A1 Single pore geometry, hollow sphere model.

strains are

$$\varepsilon_r = -P[(1-2\mu) - \theta'(1+\mu)]/[E(1-\theta)] \quad \text{(A3)}$$

$$\varepsilon_t = -P[(1-2\mu) - (\theta'/2)(1+\mu)]/[E(1-\theta)] \quad \text{(A4)}$$

where E and μ are Young's modulus and Poisson's ratio of the material, respectively.

Appendix 2. Effective bulk modulus of the hollow sphere

Effective bulk modulus (K^*) of a multiphase material is expressed as [4, 5]

$$K^* = \overline{\sigma_{ii}}/3\overline{\varepsilon_{ii}} \quad \text{(A5)}$$

where $\overline{\sigma_{ii}}$ and $\overline{\varepsilon_{ii}}$ are averages of the hydrostatic components of the stress and strain. For the hollow sphere model, the terms $\overline{\sigma_{ii}}$ and $\overline{\varepsilon_{ii}}$ can be expressed as,

$$\overline{\sigma_{ii}} = \int_{V_m}(\sigma_r + 2\sigma_t)\mathrm{d}V/V_0 + \int_{V_p}(\sigma_r + 2\sigma_t)\mathrm{d}V/V_0 \quad \text{(A6)}$$

$$\overline{\varepsilon_{ii}} = \int_{V_m}(\varepsilon_r + 2\varepsilon_t)\mathrm{d}V/V_0 + \int_{V_p}(\varepsilon_r + 2\varepsilon_t)\mathrm{d}V/V_0 \quad \text{(A7)}$$

where V_m and V_p correspond to volume of the material and the void, respectively, and V_0 is the total volume. Using Equations A1 to A4, each term in Equations A6 and A7 can be determined as follows

$$\int_{V_m}(\sigma_r + 2\sigma_t)\mathrm{d}V/V_0 = -3P \quad \text{(A8)}$$

$$\int_{V_m}(\varepsilon_r + 2\varepsilon_t)\mathrm{d}V/V_0 = -3P(1-2\mu)/E \quad \text{(A9)}$$

where P is the externally applied pressure, μ is Poisson's ratio of the material and E is Young's modulus. As the stress is zero inside the void

$$\int_{V_p} (\sigma_r + 2\sigma_t)\mathrm{d}V/V_0 = 0 \qquad (A10)$$

The second term of the right-hand side of Equation A7 can be written as

$$\int_{V_p} (\varepsilon_r + 2\varepsilon_t)\mathrm{d}V/V_0 = \int_{V_p} [(\varepsilon_r + 2\varepsilon_t)\mathrm{d}V/V_p]\,(V_p/V_0) \qquad (A11)$$

where

$$\int_{V_p} (\varepsilon_r + 2\varepsilon_t)\mathrm{d}V/V_p = \text{volume strain of the void}$$
$$= 3\Delta b/b \qquad (A12)$$

where b is internal radius of the hollow sphere and Δb is the displacement at $r = b$ under the external pressure, P.

As the tangential strain $\varepsilon_t = u/r$, where u is the displacement at a distance r from the centre of the sphere

$$\Delta b/b = (\varepsilon_t)_{r=b} \qquad (A13)$$

Combining Equations A11 to A14 we obtain

$$\int_{V_p} (\varepsilon_r + 2\varepsilon_t)\mathrm{d}V/V_0 = -9P\theta(1-\mu)/[2E(1-\theta)] \qquad (A14)$$

where $\theta = V_p/V_0$

Finally, combining Equations A5 to A13, and also using the equation expressing K in terms of E and μ we obtain

$$K^* = K(1-\theta)/(1 + b_k\theta) \qquad (A15)$$

where K is bulk modulus of the material and $b_k = (1-\mu)/[2(1-2\mu)]$.

Equation A15 can be derived more easily by simply dividing the overall volume strain ($\Delta a/a$) by the external pressure applied. But the above procedure has been provided in order to maintain continuity with the next section.

Appendix 3. Effective shear modulus

Effective shear modulus (G^*) of any multiphase material is expressed as [4, 5]

$$G^* = \overline{S_{ij}}/2\overline{e_{ij}} \qquad (A16)$$

where $\overline{S_{ij}}$ and $\overline{e_{ij}}$ are averages of the deviatoric components of the stress and strain, respectively.

For the hollow sphere, $\overline{S_{ij}}$ and $\overline{e_{ij}}$ simplify to

$$\overline{S_r} = \overline{\sigma_r - (\sigma_r + 2\sigma_t)/3}$$
$$= (2/3)(\overline{\sigma_r - \sigma_t}) \qquad (A17)$$

$$\overline{S_t} = \overline{\sigma_t - (\sigma_r + 2\sigma_t)/3}$$
$$= -(1/3)(\overline{\sigma_r - \sigma_t}) \qquad (A18)$$

$$\overline{e_r} = \overline{\varepsilon_r - (\varepsilon_r + 2\varepsilon_t)/3}$$
$$= (2/3)(\overline{\varepsilon_r - \varepsilon_t}) \qquad (A19)$$

$$\overline{e_t} = \overline{\varepsilon_t - (\varepsilon_r + 2\varepsilon_t)/3}$$
$$= -(1/3)(\overline{\varepsilon_r - \varepsilon_t}) \qquad (A20)$$

Substituting Equations A17 to A20 in to Equation A16 we get

$$2G^* = \overline{S_r}/\overline{e_r} = \overline{S_t}/\overline{e_t} = (\overline{\sigma_r - \sigma_t})/(\overline{\varepsilon_r - \varepsilon_t}) \qquad (A21)$$

Let us consider the numerator of the right-hand side of Equation A21 separately

$$\sigma_r - \sigma_t = \int_{V_m} (\sigma_r - \sigma_t)\mathrm{d}V/V_0 + \int_{V_p} (\sigma_r - \sigma_t)\mathrm{d}V/V_0 \qquad (A22)$$

where V_m and V_p are the volumes of the material and the void, respectively, V_0 is the total volume. σ_r and σ_t, from Equations A1 and A2, are substituted in the first term of the right-hand side of Equation A22. The second term, being an integral over the void volume, becomes zero. This results in

$$\overline{\sigma_r - \sigma_t} = \left[\int_b^a \frac{3Pb^3}{2(1-\theta)r^3}\,4\pi r^2\mathrm{d}r\right] + \left[\frac{4\pi r^2}{3}\right]$$

simplifying to

$$\overline{\sigma_r - \sigma_t} = \frac{-3P\theta\ln\theta}{2(1-\theta)} \qquad (A23)$$

Now considering the denominator of Equation A21

$$\overline{\varepsilon_r - \varepsilon_t} = \int_{V_m} (\varepsilon_r - \varepsilon_t)\mathrm{d}V/V_0 + \int_{V_p} (\varepsilon_r - \varepsilon_t)\mathrm{d}V/V_0 \qquad (A24)$$

ε_r and ε_t are substituted from Equations A3 and A4 in the first term of the right-hand side of Equation A24. The second term being an unknown function, it is denoted as Φ. Then

$$\overline{\varepsilon_r - \varepsilon_t} = \frac{-3P(1+\mu)\theta\ln\theta}{2E(1-\theta)} + \Phi \qquad (A25)$$

Substituting Equations A23 and A25 into Equation A21, and considering the relationship between E, G and μ we get

$$G^* = \frac{G3P\theta\ln\theta}{3P\theta\ln\theta - 4G(1-\theta)\Phi} \qquad (A26)$$

In Equation A26, Φ is an unknown function corresponding to the average of the shear strain of the void geometry which cannot be determined by normal procedures. Therefore, an additional independent equation is required to resolve Φ. This can be achieved by assuming a particular kind of variation of the effective Poisson's ratio as a function of porosity.

The effective Poisson's ratio is given by the equation

$$\mu^* = \frac{3K^* - 2G^*}{6K^* + 2G^*} \qquad (A27)$$

Substituting K^* from Equation A15 and G^* from Equation A26 into Equation A27 and also using the inter-relationships between K, G and μ we get

$$-(1-\theta)\Phi = \frac{9P\theta\ln\theta}{8G(1-\theta)} \times \left[\frac{2(\mu^* - \mu)}{(1+\mu)(1-2\mu^*)} + \frac{\theta(1-\mu^*)}{(1-2\mu^*)}\right] \qquad (A28)$$

Consider a hypothetical case where the porosity becomes unity ($\theta = 1$) and the effective Poisson's ratio for this case is assumed to be independent of the material and the value of it to be k for any value of μ and G. With this

$$-(1 - \theta)\,\Phi = \frac{-9P\,(1 + k)\,(1 - \mu)}{8G\,(1 - 2k)(1 + \mu)} \tag{A29}$$

Also consider a case where $\mu \rightarrow k$ and $\mu^* \rightarrow k$ for any value of θ

$$-(1 - \theta)\Phi = \frac{9P\,(\theta^2 \ln \theta)(1 - k)}{8G\,(1 - \theta)(1 - 2k)} \tag{A30}$$

The general function $\Phi(\theta, \mu)$ should satisfy both Equations A29 and A30 and it is found to be

$$-(1 - \theta)\Phi = \frac{9P\,(\theta^2 \ln \theta)(1 + k)(1 - \mu)}{8G\,(1 - \theta)(1 - 2k)(1 + \mu)} \tag{A31}$$

Finally, substituting Φ from Equation A31 into Equation A26 we obtain

$$G^* = \frac{G(1 - \theta)}{(1 + b_g\theta)}$$

where

$$b_g = \frac{1 + 7k - 5\mu + k\mu}{2(1 - 2k)(1 + \mu)}$$

k was found to be 1/4 from experimental results. For $k = 1/4$, $b_g = 11 - 19\mu/[4(1 + \mu)]$.

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